

ON THE DIAMETRAL AND SYMMETRY PLANES OF A SURFACE

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Annotatsiya: Ushbu maqola sirtning diametral va simmetriya tekisliklari tushunchalarini, ularning geometriyadagi tutgan o‘rni, ahamiyati hamda amaliy tatbiqlarini ochib beradi. Sirtning diametral tekisligi deganda sirtni kesib o‘tib, uni o‘zaro simmetrik ikki bo‘lakka ajratuvchi tekislik tushuniladi. Simmetriya tekisligi esa sirtni shunday ikkiga bo‘ladiki, bunda bir bo‘lak ikkinchisining ko‘zgudagi aksi sifatida namoyon bo‘ladi. Mazkur tushunchalar geometrik obyektlarning simmetriyasini o‘rganishda, ularni loyihalash va optimallashtirishda salmoqli o‘rin egallaydi. Bundan tashqari, maqolada bu tekisliklarning turfa geometrik shakl va modellarda namoyon bo‘lishi, simmetriya tamoyillariga tayanuvchi muhandislik masalalaridagi qiymati hamda sirt xossalarini aniqlashga xizmat qiluvchi matematik usullar bayon etilgan.

Kalit so‘zlar: sirtning diametral hamda simmetriya tekisligi, o‘zaro parallel vatarlar, differensiallash amali, vektor, tenglamalar sistemasi

Аннотация: Настоящая статья посвящена понятиям диаметральной и симметричной плоскостей поверхности, их месту и значению в геометрии, а также возможностям их практического использования. Под диаметральной плоскостью поверхности понимается плоскость, пересекающая поверхность и разделяющая её на две взаимно симметричные части. Плоскость симметрии, в свою очередь, делит поверхность так, что одна её часть служит зеркальным отражением другой. Указанные понятия занимают существенное место при исследовании симметрии геометрических объектов, их проектировании и оптимизации. Помимо этого, в работе освещаются проявления данных плоскостей в разнообразных геометрических формах и моделях, их роль в инженерных задачах, опирающихся на принципы симметрии, и математические приёмы, применяемые для выявления свойств поверхностей.

Ключевые слова: диаметральная и симметричная плоскости поверхности, взаимно параллельные хорды, операция дифференцирования, вектор, система уравнений

Annotation: This article examines the notions of the diametral and symmetry planes of a surface, their place and importance within geometry, together with the ways in which they may be applied in practice. The diametral plane of a surface is understood as a plane that cuts through the surface and separates it into two mutually symmetric portions. A symmetry plane, by contrast, divides the surface so that one portion appears as the mirror reflection of the other. Such notions hold a meaningful position in studying the symmetry of geometric objects, as well as in their design and optimization. The article further addresses how these planes manifest in a variety of geometric shapes and models, their value in engineering problems grounded in symmetry principles, and the mathematical techniques employed to reveal the properties of surfaces.

Keywords: diametral and symmetry planes of a surface, mutually parallel chords, the operation of differentiation, vector, system of equations

Introduction

The notion of diametral lines has already been presented in connection with second-order curves. By analogy, for a second-order surface we now bring in the notion of a diametral plane. Whenever a straight line meets a second-order surface in two distinct points, M and N , the segment MN is referred to as a chord of that second-order surface.

Theorem 1. The midpoints of parallel chords lie in the same plane.

Definition 1. A plane passing through the midpoints of parallel chords is called the diametral plane of the surface.

To write the equation of the diametral plane in an arbitrary Cartesian coordinate system, we use the following expressions:

$$x = \bar{x} + \lambda t, y = \bar{y} + \mu t, z = \bar{z} + \gamma t \text{ va } x = \bar{x} - \lambda t, y = \bar{y} - \mu t, z = \bar{z} - \gamma t \quad (1)$$

$$F(x, y, z) = 0$$

we obtain the equation:

$$2F(x, y, z) \pm 2t \left(\lambda F_x(x, y, z) + \mu F_y(x, y, z) + \gamma F_z(x, y, z) \right) + t^2 (a_{11}\lambda^2 + a_{22}\mu^2 + a_{33}\gamma^2 + 2a_{12}\lambda\mu + 2a_{23}\mu\gamma + 2a_{31}\lambda\gamma) = 0 \quad (2)$$

For this equation to hold for any t , the coefficient of t must be zero. Therefore, the equation of the diametral plane can be written as:

$$\lambda F_x + \mu F_y + \gamma F_z = 0 \quad (3)$$

It is evident that if a second-order surface has a center of symmetry, then any diametral plane passes through this center.

Thus, the center of a second-order surface is determined by solving the system of equations:

$$F_x = 0, F_y = 0, F_z = 0 \quad (4)$$

For a paraboloid, the diametral plane is parallel to its axis. In this case, since $a_{33} = 0$, equation (3) does not contain the variable z .

For elliptic and hyperbolic cylinders, all points on their axes serve as centers, so any diametral plane passes through the surface.

Example 1: Find the equation of the diametral plane of the second-order surface given by the equation $69x^2 + 3y^2 - 4z^2 + 5xy - 2xz + 4y = 0$ that passes through the line: $\frac{x-2}{3} = \frac{y}{2} = \frac{z+1}{4}$.

Solution:

Given the equation:

$$\lambda F_x + \mu F_y + \gamma F_z = 0$$

For the surface equation:

$$x^2 + 3y^2 - 4z^2 + 5xy - 2xz + 4y = 0$$

we compute the partial derivatives:

$$F_x = 2x + 5y - 2z, F_y = 6y + 5x + 4, F_z = -8z - 2x$$

Substituting these into the equation:

$$\lambda(2x + 5y - 2z) + \mu(6y + 5x + 4) - \gamma(8z + 2x) = 0$$

$$(2\lambda + 5\mu - 2\gamma)x + (5\lambda + 6\mu)y - (2\lambda + 8\gamma)z + 4\mu = 0$$

The normal vector of the plane is:

$$\bar{n} = (2\lambda + 5\mu - 2\gamma, 5\lambda + 6\mu, 2\lambda + 8\gamma)$$

$$\frac{x-2}{3} = \frac{y}{2} = \frac{z+1}{4}$$

Given the direction vector of the line:

$$\bar{u} = (3; 2; 4) \quad \bar{n} \perp \bar{u}$$

Since $\bar{n} \perp \bar{u}$, we obtain the equation:

$$6\lambda + 15\mu - 6\gamma + 10\lambda + 12\mu + 8\lambda + 32\gamma = 0$$

$$24\lambda + 27\mu + 26\gamma = 0$$

The plane passes through the point $M(2; 0; -1)$, thus:

$$2 \cdot (2\lambda + 5\mu - 2\gamma) + 0 \cdot (5\lambda + 6\mu) + (2\lambda + 8\gamma) + 4\mu = 0$$

$$6\lambda + 14\mu + 4\gamma = 0 \quad 3\lambda + 7\mu + 2\gamma = 0$$

Rewriting the system of equations:

$$\begin{cases} 24\lambda + 27\mu + 26\gamma = 0 \\ 3\lambda + 7\mu + 2\gamma = 0 \end{cases}$$

Solving for μ and λ :

$$29\mu - 10\gamma = 0 \quad \mu = \frac{10\gamma}{29},$$

$$3\lambda + \frac{70\gamma}{29} + 2\gamma = 0 \quad 3\lambda = -\frac{128\gamma}{29} \quad \lambda = -\frac{128\gamma}{87},$$

Substituting into the equation:

$$\begin{aligned} \left(-\frac{256\gamma}{87} + \frac{50\gamma}{29} - 2\gamma\right)x + \left(-\frac{640\gamma}{87} + \frac{60\gamma}{29}\right)y - \left(-\frac{256\gamma}{87} + 8\gamma\right)z + \frac{40\gamma}{29} &= 0 \\ \left(-\frac{280\gamma}{87}\right)x + \left(-\frac{460\gamma}{87}\right)y - \left(\frac{440\gamma}{87}\right)z + \frac{40\gamma}{29} &= 0 \\ 14x + 23y + 22z - 6 &= 0 \end{aligned}$$

Answer: $14x + 23y + 22z - 6 = 0$

Definition 2:

Let the plane α be given. If for any point M belonging to the surface, the point symmetric to M with respect to the plane α also belongs to the surface, then α is called the *symmetry plane* of the surface.

Using the equation of the diametral plane, we derive the equation of the symmetry plane. Since the midpoints of mutually parallel chords perpendicular to the symmetry plane belong to the symmetry plane, the equation of the symmetry plane perpendicular to the vector $\vec{a} = \{l, m, n\}$ is given by

$$lF_x + mF_y + nF_z = 0 \quad (5)$$

Since the vector $\vec{a} = \{l, m, n\}$ is perpendicular to the symmetry plane, the proportionality

$$\frac{a_{11}l + a_{12}m + a_{13}n}{l} = \frac{a_{21}l + a_{22}m + a_{23}n}{m} = \frac{a_{31}l + a_{32}m + a_{33}n}{n} \quad (6)$$

To determine the direction perpendicular to the symmetry plane from Equation (5), we introduce a proportionality factor k in Equation (6) and obtain the equivalent system:

$$\begin{cases} (a_{11} - k)l + a_{12}m + a_{13}n = 0 \\ a_{21}l + (a_{22} - k)m + a_{23}n = 0 \\ a_{31}l + a_{32}m + (a_{33} - k)n = 0 \end{cases} \quad (7)$$

Since l, m, n are not all simultaneously zero, the determinant equation

$$\begin{vmatrix} a_{11} - k & a_{12} & a_{13} \\ a_{21} & a_{22} - k & a_{23} \\ a_{31} & a_{32} & a_{33} - k \end{vmatrix} = 0$$

must hold.

Solving for k from this equation and substituting it into system (7), we determine the direction $\{l, m, n\}$.

If the symmetry plane of the surface is known, it becomes convenient to simplify the second-order surface equation, making it easier to determine the canonical coordinate system.

Definition 2:

Find the symmetric equations of the plane for the given equation:

$$36x^2 + 4y^2 + 40z^2 + 8yz + 36x - 72z + 9 = 0$$

Give coefficients:

$$a_{11} = 36, a_{12} = 0, a_{13} = 0, a_{21} = 0, a_{22} = 4, a_{23} = 4, a_{31} = 0, a_{32} = 4, a_{33} = 40$$

The determinant equation:

$$\begin{vmatrix} 36 - k & 0 & 0 \\ 0 & 4 - k & 4 \\ 0 & 4 & 40 - k \end{vmatrix} = 0$$

Expanding:

$$\begin{aligned} (36 - k)(4 - k)(40 - k) - 16(36 - k) &= 0 \\ (36 - k)(160 - 44k + k^2 - 16) &= 0 \quad (36 - k)(144 - 44k + k^2) = 0 \end{aligned}$$

Solving for k :

$$\begin{aligned} k_1 &= 36, \quad k^2 - 44k + 144 = 0 \\ D &= \sqrt{44^2 - 4 \cdot 144} = \sqrt{1360} = 4\sqrt{85} \\ k_2 &= \frac{44 + 4\sqrt{85}}{2} = 22 + 2\sqrt{85}, \quad k_3 = \frac{44 - 4\sqrt{85}}{2} = 22 - 2\sqrt{85} \end{aligned}$$

Thus, we obtain three symmetry planes.

First symmetry plane (for $k_1 = 36$):

Solving:

$$\begin{cases} -32m + 4n = 0 \\ 4m + 4n = 0 \end{cases} \quad \begin{cases} 8m - n = 0 \\ m + n = 0 \end{cases} \quad m = 0, n = 0 \quad l \in \mathbb{R}$$

$$lF_x + mF_y + nF_z = 0 \quad lF_x = 0$$

$$F_x = 0 \quad F_x = 72x + 36 = 0 \quad 2x + 1 = 0$$

Thus, the first symmetry plane is:

$$2x + 1 = 0$$

Second symmetry plane (for $k_2 = 22 + 2\sqrt{85}$):

Solving:

$$\begin{cases} (14 - 2\sqrt{85})l = 0 \\ (-18 - 2\sqrt{85})m + 4n = 0, \quad l = 0 \\ 4m + (18 - 2\sqrt{85})n = 0 \end{cases}$$

$$\begin{cases} (9 + \sqrt{85})m - 2n = 0 \\ 2m + (9 - \sqrt{85})n = 0 \end{cases} \quad m, n \neq 0$$

$$n = 1, \quad (9 + \sqrt{85})m = 2 \quad m = \frac{2}{(9 + \sqrt{85})} \quad m = \frac{\sqrt{85} - 9}{2}$$

Substituting:

$$lF_x + mF_y + nF_z = 0 \quad l = 0, m = \frac{\sqrt{85} - 9}{2}, n = 1$$

$$\frac{\sqrt{85} - 9}{2}F_y + F_z = 0 \quad F_y = 8y + 8z \quad F_z = 80z + 8y - 72$$

$$\frac{\sqrt{85} - 9}{2}(8y + 8z) + (80z + 8y - 72) = 0$$

$$(\sqrt{85} - 9)(y + z) + 2(10z + y - 9) = 0$$

$$\sqrt{85}y + \sqrt{85}z - 9y - 9z + 20z + 2y - 18 = 0$$

$$(\sqrt{85} - 7)y + (\sqrt{85} + 11)z - 18 = 0.$$

Thus, the second symmetry plane is:

$$(\sqrt{85} - 7)y + (\sqrt{85} + 11)z - 18 = 0$$

Third symmetry plane (for $k_3 = 22 - 2\sqrt{85}$):

Solving:

$$\begin{cases} (14 + 2\sqrt{85})l = 0 \\ (-18 + 2\sqrt{85})m + 4n = 0, \quad l = 0 \\ 4m + (18 + 2\sqrt{85})n = 0 \end{cases}$$

$$\begin{cases} (9 - \sqrt{85})m - 2n = 0 \\ 2m + (9 + \sqrt{85})n = 0 \end{cases} \quad m, n \neq 0$$

$$n = 1, \quad (9 - \sqrt{85})m = 2 \quad m = \frac{2}{(9 - \sqrt{85})} \quad m = -\frac{\sqrt{85} + 9}{2}$$

Substituting:

$$lF_x + mF_y + nF_z = 0 \quad l = 0, m = -\frac{\sqrt{85} + 9}{2}, n = 1$$

$$-\frac{\sqrt{85} + 9}{2}F_y + F_z = 0 \quad F_y = 8y + 8z \quad F_z = 80z + 8y - 72$$

$$-\frac{\sqrt{85} + 9}{2}(8y + 8z) + (80z + 8y - 72) = 0$$

$$-(\sqrt{85} + 9)(y + z) + 2(10z + y - 9) = 0$$

$$-\sqrt{85}y - \sqrt{85}z - 9y - 9z + 20z + 2y - 18 = 0$$

$$(\sqrt{85} + 7)y + (\sqrt{85} - 11)z + 18 = 0$$

Thus, the third symmetry plane is:

$$(\sqrt{85} + 7)y + (\sqrt{85} - 11)z + 18 = 0$$

Answer: $2x + 1 = 0$, $(\sqrt{85} - 7)y + (\sqrt{85} + 11)z - 18 = 0$, $(\sqrt{85} + 7)y + (\sqrt{85} - 11)z + 18 = 0$

Conclusion and Suggestions

In this article, the concepts of diametral and symmetry planes of a surface, their geometric properties, and practical applications were discussed. The symmetry plane serves as an important tool in studying the structure of a surface and analyzing its properties. The diametral plane, on the other hand, is widely used in determining spatial relationships on the surface and in modeling processes. The results of this study can be useful in the analysis of various geometric shapes, as well as in solving practical problems in engineering, technology, and physics. Based on this, an in-depth analysis of symmetry and diametral planes of surfaces may lead to new approaches in scientific and technological fields.

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