THE PURSUIT PROBLEM OF CAPTURING AN OBJECT FOR SECOND-ORDER CONTROLS WITH GRONUOLL-TYPE CONSTRAINTS

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DOI: ***https://doi.org/10.5281/zenodo.21059269***

Abstract. This article studies the capture (pursuit) problem for second-order differential games in the case where the controls are subject to Gronuoll-type constraints. A parallel pursuit strategy is constructed for the pursuer, and with its help, sufficient conditions for solving the capture problem are presented.

Keywords: capture, differential game, geometric constraint, parallel pursuit strategy, pursuer, evader, acceleration, Gronuoll-type constraint

GRONUOLL CHEGARALANISHLI IKKINCHI TARTIBLI BOSHQARUVLAR UCHUN QUVLOVCHINING OBYEKTNI TUTISH MASALASI

Annotatsiya: Ushbu maqolada boshqaruvlar Gronuoll chegaralanishga ega bo'lgan hol uchunhamda ikkinchi tartibli differensial o'yinlar uchun tutish masalasi o'rganiladi. Bunda quvlovchi uchun parallel quvish strategiyasi quriladi va uning yordamida tutish masalasi uchun yetarli shartlar keltiriladi.

Kalit so'zlar: Tutish, differensial o'yin, geometrik chegaralanish, parallel quvish strategiyasi, quvlovchi, qochuvchi, tezlanish, Gronuoll chegaralanishli

Let $\mathbf{P}$ and $\mathbf{E}$ objects with opposite aim be given in $\boldsymbol{R}^n$ space and their movements based on the following differential equations and initial conditions

$$\mathbf{P}: \ddot{x} = u,\ x_1 - kx_0 = 0,\ |u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds, \qquad (1)$$

$$\mathbf{E}: \ddot{y} = v,\ y_1 - ky_0 = 0,\ |v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\, ds, \qquad (2)$$

where $x, y, u, v \in \boldsymbol{R}^n$; x – a position of **P** object in $\boldsymbol{R}^n$ space, $x_0 = x(0)$, $x_1 = \dot{x}(0)$ – its initial position and velocity respectively at $t = 0$; u – a controlled acceleration of the pursuer, mapping $u : [0, \infty) \to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $u(\cdot)$ such that satisfies the condition $|u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2\, ds$ by G_P. y – a position of **E** object in $\boldsymbol{R}^n$ space, $y_0 = y(0)$, $y_1 = \dot{y}(0)$ – its initial position and velocity respectively at $t = 0$; v – a controlled acceleration of the evader, mapping $v : [0, \infty) \to \boldsymbol{R}^n$ and it is chosen as a measurable function with respect to time; we denote a set of all measurable functions $v(\cdot)$ such that satisfies the condition $|v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2\, ds$ by G_E.

Definition 1. For a trio of $(x_0, x_1, u(\cdot)), u(\cdot) \in G_P$, the solution of the equation (1), that is, $x(t) = x_0 + x_1 t + \int_0^t\int_0^s u(\tau)d\tau ds$ is called a trajectory of the pursuer on interval $t \ge 0$.

Definition 2. For a trio of $(y_0, y_1, v(\cdot)), v(\cdot) \in G_E$, the solution of the equation (2), that is, $y(t) = y_0 + y_1 t + \int_0^t\int_0^s v(\tau)d\tau ds$ is called a trajectory of the evader on interval $t \ge 0$.

Definition 3. The pursuit problem for the differential game (1) - (2) is called to be solved if there exists such control function $u^*(\cdot) \in \boldsymbol{G_p}$ of the pursuer for any control function $v(\cdot) \in \boldsymbol{G_E}$ of the evader and the following equality is carried out at some finite time t^*

$$x(t^*) = y(t^*). \qquad (3)$$

Definition 4. For the problem (1)-(2), time T is called a guaranteed pursuit time if it is equal to an upper boundary of all the finite values of pursuit time t^* which the equality (3) is true[2-4].

Definition 5. For the differential game (1) - (2), the following function is called П-strategy of the pursuer ([3]-[4]):

$$u(v) = v - \lambda(v)\xi_0, \qquad (4)$$

where $\xi_0 = \dfrac{z_0}{|z_0|}$, $\lambda(v) = (v, \xi_0) + \sqrt{(v, \xi_0)^2 + \delta e^{2lt}}$, $\delta = \rho^2 - \sigma^2 \ge 0$,

(v, ξ_0) is a scalar multiplication of vectors v and ξ_0 in the space $\boldsymbol{R^n}$.

Lemma 1 (Granwoll). Suppose, let a mapping $\varphi(t):[0,\infty) \to \boldsymbol{R^n}$ be bounded, nonnegative and measurable function. Moreover, $l \ge 0$ and $\rho > 0$ are constant and for the given if an inequality $|\varphi(t)|^2 \le \rho^2 + 2l\int_0^t |\varphi(s)|^2\, ds$ is carried out, then a relation $\varphi(t) \le \rho e^{lt}$ is always true.

Lemma 2 . If $\rho \ge \sigma$, then the following inequality is true for the function $\lambda(v, \xi_0)$:

$$e^{lt}(\rho - \sigma) \le \lambda(v, \xi_0) \le e^{lt}(\rho + \sigma).$$

Theorem. If for the second order differential game (1) – (2) with Granwoll constraint a condition $\rho > \sigma$ is true, then the pursuit problem is solved by П-strategy (4) on interval $(0, t)$ and an approach function between the objects becomes as follows:

$$f(l, t, |z_0|, \rho, \sigma, k) = |z_0|(kt + 1) - \frac{\rho - \sigma}{l^2} e^{lt} + \frac{\rho - \sigma}{l^2} + \frac{\rho - \sigma}{l} t$$

Proof. Suppose, let the pursuer choose a strategy in the form (4) when the evader chooses any control function $v(\cdot)\in \boldsymbol{G}_{\boldsymbol{E}}$. Then according to the equations (1) and (2) we define the following Caratheodory’s equation

$$\ddot{z} = -\lambda\left(v\left(t\right)\right)\xi_0, \qquad \dot{z}\left(0\right) - kz\left(0\right) = 0,$$

Hence the following solution will be found by the given initial conditions

$$z\left(t\right) = z_0\left(kt+1\right) - \xi_0 \int_0^t\int_0^s \lambda\left(v\left(\tau\right),\xi_0\right)d\tau ds$$

or

$$\left|z\left(t\right)\right| = \left|z_0\right|\left(kt+1\right) - \int_0^t\int_0^s ((v,\xi_0) + \sqrt{(v,\xi_0)^2 + \delta e^{2lt}})d\tau ds \,.$$

We form the following inequalities in relation to **Lemma 1**

$$\left|z\left(t\right)\right| \le \left|z_0\right|\left(kt+1\right) - \int_0^t\int_0^s e^{l\tau}(\rho-\sigma)d\tau ds \Rightarrow$$

$$\left|z(t)\right| \le \left|z_0\right|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

If we say $f\,(l,t,\left|z_0\right|,\rho,\sigma,k) = \left|z_0\right|(kt+1) - \dfrac{\rho-\sigma}{l^2}e^{lt} + \dfrac{\rho-\sigma}{l^2} + \dfrac{\rho-\sigma}{l}t$ (5), then define a positive solution t^* such that the function (5) equals to zero

$$\frac{\rho-\sigma}{l^2}e^{lt} = \left|z_0\right|(kt+1) + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t\,.$$

We will form the following equality by simplifying the latest relation

$$e^{lt} = t\left(\frac{\left|z_0\right|kl^2}{\rho-\sigma} + l\right) + \frac{\left|z_0\right|l^2}{\rho-\sigma} + 1$$

where $A = \dfrac{\left|z_0\right|kl^2}{\rho-\sigma} + l$, $B = \dfrac{\left|z_0\right|l^2}{\rho-\sigma} + 1$, $B > 1$. Therefore, we have the following equation

$$e^{lt} = At + B \qquad (6)$$

In order to define a pursuit time we will consider some cases of the equation (5).

1. Let be $A < 0 \Rightarrow k < \dfrac{\sigma-\rho}{\left|z_0\right|l}$. Then the equation (5) has a unique positive solution t^* and this solution is a pursuit time (Fig-1)[3-6].

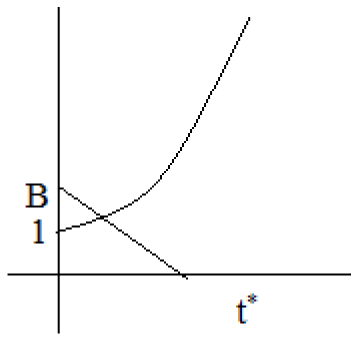

Figure-1

2. Let be $A = 0 \Rightarrow k = \dfrac{\sigma - \rho}{|z_0| l}$. Then a solution of the equation (5) is $t^* = \dfrac{\ln\left(\dfrac{|z_0| l^2}{\rho - \sigma} + 1\right)}{l}$, and this solution is a pursuit time (Fig-2).

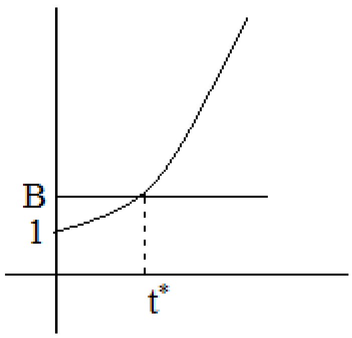

Figure-2

3. Let be $A > 0 \Rightarrow k > \dfrac{\sigma - \rho}{|z_0| l}$. Then the equation (5) has a positive solution t^* and this solution is a pursuit time (Fig-3).

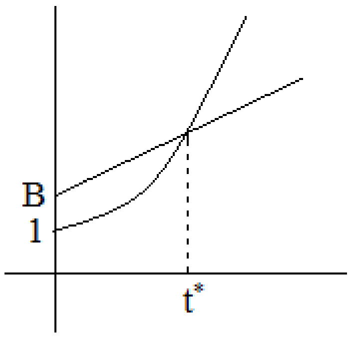

Figure-3

In conclusion, the relation (3) is true in all values of interval $t \geq 0$ according to the inequality $|z(t)| \leq f(k, t, \rho, \sigma, l, |z_0|)$ and properties of (5), i.e., the evasion problem is solved. Proved.

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